## Relationship of cloud flare ups, growth of divergence, enhancement of the clepartures firom balance laws and hurricane intensity

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## ARW Model Description

The real-time ARW forecasts in 2005 used a two-way nested configuration (Michalakes et al. 2005), that featured a $12-\mathrm{km}$ outer fixed domain with a movable nest of $4 / 1.33-\mathrm{km}$ grid spacing.
The nest was centered on the location of the minimum $500-\mathrm{hPa}$ geopotential height within a prescribed search radius from the previous position of the vortex center (or within a radius of the first guess, when first starting).
Nest repositioning was calculated every 15 simulation minutes and the width of the search radius was based on the maximum distance the vortex could move at $40 \mathrm{~m} \mathrm{~s}^{-1}$.
On the 12-km domain, the Kain-Fritsch cumulus parameterization was used, but domains with finer resolution had no parameterization.
All domains used the WRF single-moment 3-class (WSM3) microphysics scheme (Hong et al. 2004) that predicted only one cloud variable (water for $T>0^{\circ} \mathrm{C}$ and ice for $T<0^{\circ} \mathrm{C}$ ) and one hydrometeor variable, either rainwater or snow (again thresholded on $0^{\circ} \mathrm{C}$ ).

Both domains also used the Yonsei University (YSU) scheme for the planetary boundary layer (Noh et al. 2003).
This is a first-order closure scheme that is similar in concept to the scheme of Hong and Pan (1996), but appears less biased toward excessive vertical mixing as reported by Braun and Tao (2000).

The drag formulation follows Charnock (1955) and is described more in section 5. The surface exchange coefficient for water vapor follows Carlson and Boland (1978), and the heat flux uses a similarity relationship (Skamarock et al. 2005).

The forecasts were integrated from 0000 UTC and occasionally 1200 UTC during the time when a hurricane threatened landfall within 72 h .

Forecasts were initialized using the Geophysical Fluid Dynamics Laboratory (GFDL) model, with data on a $1 / 6^{0}$ latitude-longitude grid. The Global Forecast Model (GFS) from the National Centers for Environmental Prediction (NCEP), obtained on a $1^{\circ}$ grid, was used only when the GFDL was unavailable.

Davis, C., W. Wang, S.S. Chen, Y. Chen, K. Corbosiero, M. DeMaria, J. Dudhia, G. Holland, J. Klemp, J. Michalakes, H. Reeves, R. Rotunno, C. Snyder, and Q. Xiao, 2008: Prediction of Landfalling Hurricanes with the Advanced Hurricane WRF Model. Mon. Wea. Rev., 136, 1990-2005.

## Post processing diagnostics

Here we shall be showing some post processing for WRF-ARW. This model is presently being added to our suite of mesoscale models.


Katrina, valid time = 1200 UTC 29 Aug (60-h forecast)

Predicted storm center location at indicated valid times (below) is denoted by blue star in each figure. Wind fields from AHW forecasts have been shifted to observed locations to facilitate comparison.

HWind valid times are (a) 1132 UTC 29 Aug

10-m wind from AHW real-time forecasts with contours of nearest HWind (black lines) analyses overlaid



100 km

## Size of the storm as seen by the predicted wind field at different resolutions compared to HWIND.

Shown here is $10-\mathrm{m}$ wind speed ( m $s-1$ ) from 36-h Katrina forecast valid 1200 UTC 28 Aug on (a) the 12-km grid, (b) the 4-km grid, (c) the 1.33km grid, and (d) the NOAA HWind product valid 1200 UTC 28 Aug. White ellipses in (d) are an approximate trace of the radii of maximum wind at each azimuth around the vortices in (a), (b), and (c).


Model-derived reflectivity at 3-km MSL valid 2300 UTC 28 Aug from nest with (a) 1.33-km grid increment and (b) 4-km grid increment. (c) Observed radar reflectivity composite valid between 2000 and 2100 UTC 28 Aug based on tail Doppler radar data from both the NOAA P-3 (red track) and the Naval Research Laboratory P-3 (pink track) with the Electra Doppler radar (ELDORA). The composite radar image was obtained from the RAINEX field catalog maintained by the Earth Observing Laboratory of the National Center for Atmospheric Research.
OBS composite radar

We shall next illustrate several examples of the following scenario:

1. Deep convection flares up near the eye wall, as seen from the local growth of rain water mixing ratio, liquid water mixing ratio or radar reflectivity as implied from model hydrometeors.
2. Divergence flares up
3. Departures from balance laws flare up
4. Solution of complete radial equation shows rapid growth of hurricane intensity.

## Departures from balance laws

The full divergence equation can be written in the form (from Fankhauser 1974):

$$
\begin{aligned}
& -\nabla^{2} \phi=-f\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)-2\left(\frac{\partial v}{\partial x} \frac{\partial u}{\partial y}-\frac{\partial u}{\partial x} \frac{\partial v}{\partial y}\right)+\beta u+\left(\frac{\partial \omega}{\partial x} \frac{\partial u}{\partial p}+\frac{\partial \omega}{\partial y} \frac{\partial v}{\partial p}\right)+D^{2}+ \\
& \left(\frac{\partial D}{\partial t}+u \frac{\partial D}{\partial x}+v \frac{\partial D}{\partial y}+\omega \frac{\partial D}{\partial p}\right)+\left(\frac{\partial F_{u}}{\partial x}+\frac{\partial F_{v}}{\partial y}\right)
\end{aligned}
$$

Red lines represent the balance equation (Haltiner and Williams 1980). The blue underlined terms denote the non linear balance which is also expressed as.

$$
\nabla^{2} \phi=f \nabla^{2} \psi+2 J\left(\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}\right)
$$

## Implication of gradient wind departures on hurricane intensity

In local cyclindrical storm centered coordinate we can write the complete radical equation of motion in the form:
$\frac{\partial V_{r}}{\partial t}+V_{\theta} \frac{\partial V_{r}}{r \partial \theta}+V_{r} \frac{\partial V_{r}}{\partial r}+\omega \frac{\partial V_{r}}{\partial p}-\frac{V_{\theta}^{2}}{r}-f V_{\theta}=-g \frac{\partial z}{\partial r}+F_{r}$ or
$\frac{V_{\theta}^{2}}{r}+f V_{\theta}-g \frac{\partial z}{\partial r}+G W D=0$ where GWD denotes the gradient wind departure.
Where $G W D=-\frac{\partial V_{r}}{\partial t}-V_{\theta} \frac{\partial V_{r}}{r \partial \theta}-V_{r} \frac{\partial V_{r}}{\partial r}-\omega \frac{\partial V_{r}}{\partial p}-F_{r}$
$V_{\theta}$ can be expressed by the relation, $\quad V_{\theta}=\frac{-f \pm \sqrt{f^{2}-\frac{4}{r}\left(G W D-g \frac{\partial z}{\partial r}\right)}}{\frac{2}{r}}$.
This denotes a local value of the tangential wind from a complete radial wind equation in the presence of gradient wind departures GWD. Note that $-g \frac{\partial z}{\partial r}$ is generally $<0$ in the inner rain area ( $\mathrm{r}<200 \mathrm{~km}$ where r is positive outward). The gradient wind, in this locale
storm centered coordinate, is given by: $V_{\theta_{z}}=\frac{-f \pm \sqrt{f^{2}+\frac{4}{r}\left(g \frac{\partial z}{\partial r}\right)}}{\frac{2}{r}}$. Note that GWD can
be $\leq 0$. Thus several possibilities exist:
If $f^{2}<\frac{4}{r}\left(G W D-g \frac{\partial z}{\partial r}\right)$ we have a nonphysical solution.

We are looking for different options for $f^{2}-\frac{4}{r}\left(G W D-g \frac{\partial z}{\partial r}\right) 0$, and its positive root.
Since $\left.g \frac{\partial z}{\partial r}\right\rangle 0$ we can write the inequality in the form, $f^{2}+\frac{4}{r} g \frac{\partial z}{\partial r}>\frac{4}{r} G W D$. The left hand side is essentially positive definite, GWD $<0$ would always satisfy this condition. A negative value of GWD contributes to a value of $V_{\theta}$ in equation 0 larger than the gradient wind value, i.e. $\left.V_{\theta}\right\rangle V_{\text {of }}$, hence the extreme negative values of GWD would go with large values of azimuthal motions i.e supergradient winds or non physical situations. A positive value of GWD, conversely describes situations with subgradient winds. Such instances of very strong azimuthal motions described by the complete radial equation of motion, are attributed to the departure from gradient wind (GWD) which also relate to large values of divergence, this we illustrate below.

## A note on Radial gradient wind solution:

For a hurricane in storm centered coordinate the radial gradient wind equation is expressed by $\frac{V_{\theta}{ }^{2}}{r}+f V_{\theta}=g \frac{\partial z}{\partial r}$ where $\frac{V_{\theta}{ }^{2}}{r}, \mathrm{f} V_{\theta}$ and $g \frac{\partial z}{\partial r}$ are all generally positive. The roots of this equation are $u=\frac{-f \pm \sqrt{\frac{4}{r} g \frac{\partial z}{\partial r}}}{\frac{2}{r}}$. Note that the negative root of the radical is nonphysical hence there is only one positive root i.e. $\frac{-f+\sqrt{\frac{4}{r} g \frac{\partial z}{\partial r}}}{\frac{2}{r}}$ for $\left|\sqrt{\left.\frac{4}{r} g \frac{\partial z}{\partial r} \right\rvert\,}\right||f|$.

This implies that $\sqrt{\frac{4}{r} g \frac{\partial z}{\partial r}}>f$ applies for cyclonic motions. Note that there are no anomalous solutions of the radical gradient wind equation, i.e. there is only one normal
solution. Such instances of very strong azimuthal motions, described by the complete radical equation of motion, are attributed to the departures from gradient wind (GWD) which relate to large values of divergence.

The full divergence equation can be written in the form (from Fankhauser 1974):
$-\nabla^{2} \phi=-f\left(\frac{\partial V_{\theta}}{\partial y}-\frac{\partial V_{r}}{\partial x}\right)-2\left(\frac{\partial V_{r}}{\partial x} \frac{\partial V_{\theta}}{\partial y}-\frac{\partial V_{\theta}}{\partial x} \frac{\partial V_{r}}{\partial y}\right)+\beta u+\left(\frac{\partial \omega}{\partial x} \frac{\partial V_{\theta}}{\partial p}+\frac{\partial \omega}{\partial y} \frac{\partial V_{r}}{\partial p}\right)+D^{2}+$
$\left(\frac{\partial D}{\partial t}+V_{\theta} \frac{\partial D}{\partial x}+V_{r} \frac{\partial D}{\partial y}+\omega \frac{\partial D}{\partial p}\right)+\left(\frac{\partial F_{V_{\theta}}}{\partial x}+\frac{\partial F_{V_{r}}}{\partial y}\right)$
Red lines represent the balance equation (Haltiner and Williams 1980). The underlined terms denote the non linear balance which is also expressed as $\nabla^{2} \phi=f \nabla^{2} \psi+2 J\left(\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}\right)$.

The complete radial equation of motion in storm centered coordinate (r positive outward) is written in the form:
$\frac{d V_{r}}{d t}-\left(f+\frac{V_{\theta}}{r}\right) V_{\theta}=-\frac{\partial \phi}{\partial r}+g \frac{\partial \tau_{\theta}}{\partial p}$
Or $\frac{\partial V_{r}}{\partial t}=-V_{\theta} \frac{\partial V}{r \partial \theta}-V_{r} \frac{\partial V_{r}}{\partial r}-\omega \frac{\partial V_{r}}{\partial p}-\frac{V_{\theta}^{2}}{r}-f V_{\theta}=-g \frac{\partial z}{\partial r}+F_{r}$, the underlined terms denote radial gradient wind balance. The non linear balance and the radial gradient wind equation are equivalent, Fortak (1956).

Fortak, H., "Concerning the general vertically averaged hydrodynamic equations NHRP report no. 51, U.S. Weather Bureau, Washington, DC, Apr. 1962.

Thus departures from non linear balance can be approximately equated to departures from gradient wind balance for the radial direction. Departures from non linear balance largely arise from horizontal and vertical advection of divergence and the divergence square. If a flare up of deep convection occurs near the eye wall of a hurricane divergence (/convergence) increases, so do the departures from balance laws. Growth of negative departures leads to a stronger hurricane. This follows from

$$
G W D^{\prime}=\left(\frac{\partial \omega}{\partial x} \frac{\partial u}{\partial p}+\frac{\partial \omega}{\partial y} \frac{\partial v}{\partial p}\right)+D^{2}+\left(\frac{\partial D}{\partial t}+u \frac{\partial D}{\partial x}+v \frac{\partial D}{\partial y}+\omega \frac{\partial D}{\partial p}\right)+\left(\frac{\partial F_{w}}{\partial x}+\frac{\partial F_{v}}{\partial y}\right)
$$

The complete radial equation may be written as where "GWD" denotes gradient wind departures, and GWD is expressed by the radial wind equations carries the solutions $\frac{V_{\theta}^{2}}{r}+f V_{\theta}-g \frac{\partial z}{\partial r}+G W D^{\prime}=0$
the negative root is non physical. The other root, when GWD is $\ll 0$, carries strong tangential wind for its solution. When $\mathrm{GWD}=0$ then we have a radial gradient wind balance.

$$
\frac{V_{\theta}^{2}}{r}+f V_{\theta}=g \frac{\partial z}{\partial r}
$$

We have routinely mapped the field of GWD in the intensifying and decaying phases of hurricane intensity.

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Hourly plots





Initial time: 10z 28 August 2005

Hourly plots





Initial time: 09z 28 August 2005





Initial time: 09z 28 August 2005





Initial time: 09z 28 August 2005



DEPARTURE (x $10^{\wedge}-5 \mathrm{sec}, * * 2$ ) $13 z 28$ Aug 2005


DEPARTURE (x 10^-5sec.**2)14z 28 Aug 2005


DEPARTURE (x $10^{*}-5 \mathrm{sec}, * * 2$ ) $15 z 28$ Aug 2005




13z 28August 2005
Life cycle of a cloud




16z 28August 2005


-The Global Hawk or the ER-2 leave Dryden/Edwards AFB, or the DC-8 flies into the storm
-The plane observes the storm environment from above
It transmits data to NASA GSFC via a Communications Satellite -NASA GSFC transmits the data in near real-time to FSU
-FSU interprets the data to forecast imminent Intensity changes


## Observations Required

- Radar Reflectivity
- 3-Dimensional Winds
- Pressure Altitude




## Future work

- Future work on mesoscale modeling during the hurricane season of 2009 will include the following models: HWRF(EMC), WRF/ARW (NCAR), COAMPS (NRL),GFDL(NOAA), HWRF-X (HRD), MM5 (FSU), WRF(FSU).


## THANKS

